Index Laws

Part 1
Simplifying Expressions

a $x^8 \times x^{-6}$ b $y^{-2} \times y^{-4}$ c $6p^3 \div 2p^7$ d $(2x^{-4})^3$

e $y^3 \times y^{-\frac{1}{2}}$ f $2b^{\frac{2}{3}} \times 4b^{\frac{1}{4}}$ g $x^{\frac{3}{5}} \div x^{\frac{1}{3}}$ h $a^{\frac{1}{2}} \div a^{\frac{4}{3}}$

i $p^{\frac{1}{4}} \div p^{-\frac{1}{3}}$ j $(3x^{\frac{2}{3}})^2$ k $y \times y^{\frac{5}{6}} \times y^{-\frac{3}{2}}$ l $4t^{\frac{3}{2}} \div 12t^{\frac{1}{2}}$

m $\dfrac{b^2 \times b^{\frac{1}{4}}}{b^{\frac{1}{2}}}$ n $\dfrac{y^{\frac{1}{2}} \times y^{\frac{1}{3}}}{y}$ o $\dfrac{4x^{\frac{2}{3}} \times 3x^{-\frac{1}{6}}}{6x^{\frac{3}{4}}}$ p $\dfrac{2a \times a^{\frac{3}{4}}}{8a^{-\frac{1}{2}}}$

Part 2
Find the value of x such that

a $2^x = 64$ b $5^{x-1} = 125$ c $3^{x+4} - 27 = 0$ d $8^x - 2 = 0$

e $3^{2x-1} = 9$ f $16 - 4^{3x-2} = 0$ g $9^{x-2} = 27$ h $8^{2x+1} = 16$

i $49^{x+1} = \sqrt{7}$ j $3^{3x-2} = \sqrt[3]{9}$ k $(\frac{1}{6})^{x+3} = 36$ l $(\frac{1}{2})^{3x-1} = 8$

Part 3
Solving Equations

a $2^{x+3} = 4^x$ b $5^{3x} = 25^{x+1}$ c $9^{2x} = 3^{x-3}$ d $16^x = 4^{1-x}$

e $4^{x+2} = 8^x$ f $27^{2x} = 9^{3-x}$ g $6^{3x-1} = 36^{x+2}$ h $8^x = 16^{2x-1}$

i $125^x = 5^{x-3}$ j $(\frac{1}{3})^x = 3^{x-4}$ k $(\frac{1}{2})^{1-x} = (\frac{1}{8})^{2x}$ l $(\frac{1}{4})^{x+1} = 8^x$

Part 4
Problem Solving

Q1

a The area of a rectangle is $8y^5z^7$ and its length is $4y^2z^3$. Work out its width.

b Explain why the area of a rectangle of sides $\sqrt[3]{8m^{-3}n^6}$ and $\sqrt{16m^2n^{-4}}$ is independent of m and n. What is the area?

Q2

A cyclist travels $4b^2c^{\frac{1}{2}}$ miles in $3b^2c$ hours. What is her average speed?

Q3

A disc of radius $3v^2z^{-2}$ cm is removed from a disc of radius $4v^2z^{-2}$ cm. What is the remaining area?

Q4

The kinetic energy of a body is given by

$\frac{1}{2} \times \text{mass} \times \text{velocity}^2$

Work out the kinetic energy when mass $= m$ and velocity $= 9x^{\frac{3}{4}}c^{\frac{3}{4}}$

Q5

In an electrical circuit, the total resistance of two resistors, t_1 and t_2, connected in parallel, is found by dividing the product of their resistances by the sum of their resistances. Work out the total resistance when $t_1 = 3rs^2$ ohms and $t_2 = 5rs^2$ ohms.

Index Laws Solutions

Part 1
Simplifying Expressions

a $= x^2$

b $= y^{-6}$

c $= 3p^{-4}$

d $= 8x^{-12}$

e $= y^{\frac{5}{2}}$

f $= 8b^{\frac{2}{3}+\frac{1}{4}} = 8b^{\frac{11}{12}}$

g $= x^{\frac{3}{5}-\frac{1}{3}} = x^{\frac{4}{15}}$

h $= a^{\frac{1}{2}-\frac{4}{3}} = a^{-\frac{5}{6}}$

i $= p^{\frac{1}{4}-(-\frac{1}{5})} = p^{\frac{9}{20}}$

j $= 9x^{\frac{4}{5}}$

k $= y^{1+\frac{5}{6}-\frac{3}{2}} = y^{\frac{1}{3}}$

l $= \frac{1}{3}t$

m $= b^{2+\frac{1}{4}-\frac{1}{2}} = b^{\frac{7}{4}}$

n $= y^{\frac{1}{2}+\frac{1}{3}-1} = y^{-\frac{1}{6}}$

o $= 2x^{\frac{2}{3}+(-\frac{1}{6})-\frac{3}{4}} = 2x^{-\frac{1}{4}}$

p $= \frac{1}{4}a^{1+\frac{3}{4}-(-\frac{1}{2})} = \frac{1}{4}a^{\frac{9}{4}}$

Part 2

a $2^x = 2^6$
$x = 6$

b $5^{x-1} = 5^3$
$x - 1 = 3$
$x = 4$

c $3^{x+4} = 27 = 3^3$
$x + 4 = 3$
$x = -1$

d $(2^3)^x = 2^{3x} = 2$
$3x = 1$
$x = \frac{1}{3}$

e $3^{2x-1} = 3^2$
$2x - 1 = 2$
$x = \frac{3}{2}$

f $16 = 4^2 = 4^{3x-2}$
$2 = 3x - 2$
$x = \frac{4}{3}$

g $(3^2)^{x-2} = 3^{2x-4} = 3^3$
$2x - 4 = 3$
$x = \frac{7}{2}$

h $(2^3)^{2x+1} = 2^{6x+3} = 2^4$
$6x + 3 = 4$
$x = \frac{1}{6}$

i $(7^2)^{x+1} = 7^{2x+2} = 7^{\frac{1}{2}}$
$2x + 2 = \frac{1}{2}$
$x = -\frac{3}{4}$

j $3^{3x-2} = (3^2)^{\frac{1}{3}} = 3^{\frac{2}{3}}$
$3x - 2 = \frac{2}{3}$
$x = \frac{8}{9}$

k $(6^{-1})^{x+3} = 6^{-x-3} = 6^2$
$-x - 3 = 2$
$x = -5$

l $(2^{-1})^{3x-1} = 2^{1-3x} = 2^3$
$1 - 3x = 3$
$x = -\frac{2}{3}$

Part 3
Solving Equations

a $2^{x+3} = (2^2)^x = 2^{2x}$
$x + 3 = 2x$
$x = 3$

b $5^{3x} = (5^2)^{x+1} = 5^{2x+2}$
$3x = 2x + 2$
$x = 2$

c $(3^2)^{2x} = 3^{4x} = 3^{x-3}$
$4x = x - 3$
$x = -1$

d $(4^2)^x = 4^{2x} = 4^{1-x}$
$2x = 1 - x$
$x = \frac{1}{3}$

e $(2^2)^{x+2} = (2^3)^x$
$2^{2x+4} = 2^{3x}$
$2x + 4 = 3x$
$x = 4$

f $(3^3)^{2x} = (3^2)^{3-x}$
$3^{6x} = 3^{6-2x}$
$6x = 6 - 2x$
$x = \frac{3}{4}$

g $6^{3x-1} = (6^2)^{x+2}$
$6^{3x-1} = 6^{2x+4}$
$3x - 1 = 2x + 4$
$x = 5$

h $(2^3)^x = (2^4)^{2x-1}$
$2^{3x} = 2^{8x-4}$
$3x = 8x - 4$
$x = \frac{4}{5}$

i $(5^3)^x = 5^{x-3}$
$5^{3x} = 5^{x-3}$
$3x = x - 3$
$x = -\frac{3}{2}$

j $(3^{-1})^x = 3^{x-4}$
$3^{-x} = 3^{x-4}$
$-x = x - 4$
$x = 2$

k $(2^{-1})^{1-x} = (2^{-3})^{2x}$
$2^{x-1} = 2^{-6x}$
$x - 1 = -6x$
$x = \frac{1}{7}$

l $(2^{-2})^{x+1} = (2^3)^x$
$2^{-2x-2} = 2^{3x}$
$-2x - 2 = 3x$
$x = -\frac{2}{5}$

Part 4
Problem Solving

Q1

a $2y^3z^4$

b Area = 8. Therefore the area is independent of m and n

Q2

$$\frac{4}{3c^{\frac{1}{2}}}\text{ mph}$$

Q3

$7\pi v^4 z^{-4}\text{ cm}^2$

Q4

$$\frac{81}{2}mx^{\frac{3}{2}}c^{\frac{3}{2}}$$

Q5

$$\frac{15}{8}rs^2\text{ ohms}$$

(Full worked solutions on Kerboodle Ex1.2B Q 5,6,8,11,13)